

“MR. CALCULUS” ANSWERS TO THE 2010 FORM B FREE RESPONSE QUESTIONS

AB 2

$$g(1) = 2, \quad g'(x) = \sin\left(x + \frac{1}{x}\right), \quad g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$$

(a) The graph of g has a horizontal tangent where $g'(x) = 0$: $x = 0.163$ and $x = 0.359$.

(b) The graph of g is concave down where $g''(x) < 0$, when the graph of $g''(x)$ is below the x -axis. This happens on the interval, $0.129 < x < 0.223$.

(c) $g(0.3) = g(1) - \int_{0.3}^1 g'(x)dx \approx 1.546$. The slope of g at $x = 0.3$ is $g'(0.3) \approx -0.472$.

An equation of the tangent line is $y - 1.546 = -0.472(x - 0.3)$

(d) Since $g''(x) > 0$ for $0.25 < x < 1$, the graph of g is concave up there, so the tangent line at $x = 0.3$ lies below the graph of $g(x)$.