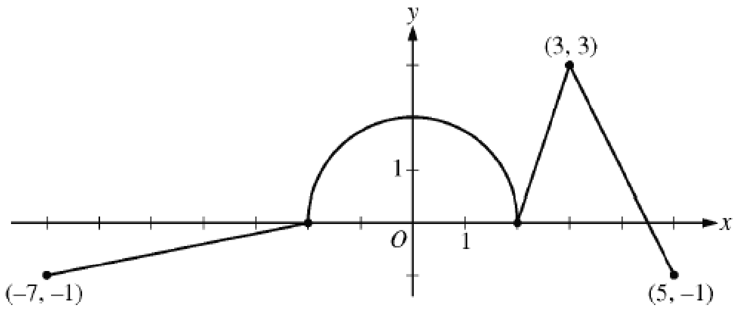
AB 5



Graph of g'

g is defined and differentiable, hence continuous, on $[-7, 5]$ and $g(0) = 5$.

(a) $g(3) = g(0) + \int_0^3 g'(x)dx = \boxed{5 + \left(\frac{1}{4}\pi(2)^2\right) + \left(\frac{1}{2}(1)(3)\right)} = 5 + \pi + \frac{3}{2} = \boxed{\frac{13}{2} + \pi}$

$g(-2) = g(0) - \int_{-2}^0 g'(x)dx = \boxed{5 - \pi}$

(b) On (–7, 5), $g'(x)$ changes from increasing to decreasing or decreasing to increasing, hence $g''(x)$ changes signs, at $\boxed{x = 0,\ x = 2, \text{ and } x = 3}$

(c) $h(x) = g(x) - \frac{1}{2}x^2$ $\qquad h'(x) = g'(x) - x = 0$ when $g'(x) = x$.

One place where this happens at $\boxed{x = 3}$.
On [2, 3], $g'(x) = 3(x-2)$ so $h'(x) = 3(x-2) - x = 2x - 6$ which is negative on [2, 3).
On [3, 5], $g'(x) = -1 - 2(x-5) = -2x + 9$ so $h'(x) = -2x + 9 - x = -3x + 9$ which is negative on (3, 5]. So, there is $\boxed{\text{neither a maximum nor a minimum at } x = 3 \text{ since } g'(x) \text{ does not change signs there}}$.

Another place where this happens is on [–2, 2] that is the part of $g'(x)$ that is the semicircle $x^2 + y^2 = 4$. So $y = g'(x) = \sqrt{4 - x^2} = x$ or $4 - x^2 = x^2 \Rightarrow \boxed{x = \sqrt{2}}$
$h''(x) = g''(x) - 1$. $\quad h''\left(\sqrt{2}\right) = g''\left(\sqrt{2}\right) - 1 < 0$ since $g''\left(\sqrt{2}\right)$ is negative because $g'(x)$ is decreasing from $x = 0$ to $x = 2$.

$\boxed{\text{Therefore } h \text{ has a relative maximum at } x = \sqrt{2} \text{ because } h'\left(\sqrt{2}\right) = 0 \text{ and } h''\left(\sqrt{2}\right) < 0}$.