AB 6

$\frac{dy}{dx} = xy^3$ and $\frac{d^2y}{dx^2} = y^3\left(1+3x^2y^2\right)$ and $f(1) = 2$

(a) At (1, 2) the slope = $\frac{dy}{dx} = (1)(2^3) = 8 \Rightarrow$ Tangent line is $\boxed{y-2=8(x-1)}$

(b) $y-2=8(x-1)$ so $y(1.1) \approx 2+8(1.1-1) = 2+0.8 = \boxed{2.8}$

$f''(1) = 2^3\left(1+3\left(1^2 2^2\right)\right) > 0$ so *f* is concave up and the tangent line lies below the graph of *f*. Hence, the approximation above is less than the actual value.

(c) $\frac{dy}{y^3} = xdx \Rightarrow -\frac{1}{2y^2} = \frac{1}{2}x^2 + C$

Since $f(1)=2$, $-\frac{1}{8} = \frac{1}{2} + C \Rightarrow C = -\frac{5}{8}$

So, $-\frac{1}{2y^2} = \frac{1}{2}x^2 - \frac{5}{8}$

Solving for y: $\frac{4}{y^2} = -4x^2+5 \Rightarrow y^2 = \frac{4}{-4x^2+5} \Rightarrow \boxed{y = \sqrt{\frac{4}{-4x^2+5}}}$

Note: The positive square root was used since $f(1) = 2 > 0$