AB 6

$p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, for $0 \le t \le 6$, and $r(t) = t^3 - 6t^2 + 9t + 3$

(a) Particle *R* moves to the right when $r'(t) = 3t^2 - 12t + 9 > 0 \Rightarrow (t-3)(t-1) > 0 \Rightarrow$ $\boxed{0 \le t < 1 \text{ and } 3 < t \le 6}$. Note: $r'(t) = 0$ when t = 3 and 1.

(b) Particle *P* moves to the right when
$p'(t) = -\frac{\pi}{2}\sin\left(\frac{\pi}{4}t\right) > 0 \Rightarrow \sin\left(\frac{\pi}{4}t\right) < 0 \Rightarrow \pi < \frac{\pi}{4}t < 2\pi \Rightarrow 4 < t \le 6.$

The particles travel in opposite direction for $\boxed{0 < t < 1 \text{ and } 3 < t < 4}$.
Note: $p'(t) = 0$ when t = 0 and 4.

(c) The acceleration is $p''(3) = -\frac{\pi^2}{8}\cos\left(\frac{3\pi}{4}\right) = -\frac{\pi^2}{8}\cdot\left(-\frac{\sqrt{2}}{2}\right) = \frac{\pi^2\sqrt{2}}{16}$. The velocity at t = 3 is $p'(3) = -\frac{\pi}{2}\sin\left(\frac{3\pi}{4}\right) = -\frac{\pi\sqrt{2}}{4} < 0$. Since the velocity and acceleration have opposite signs, the $\boxed{\text{particle is slowing down at } t = 3}$.

(d) The distance between the particles is $|p(t) - r(t)|$ so the average distance on the interval $1 \le t \le 3$ is $\boxed{\frac{1}{3-1}\int_1^3 |p(t) - r(t)|\,dt.}$