

**“MR. CALCULUS” ANSWERS TO THE 2010 FREE RESPONSE QUESTIONS**

**AB/BC 2**

$t$ (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

(a)  $E(6) \approx \frac{E(7) - E(5)}{7 - 5} = \frac{21 - 13}{2} = \boxed{4 \frac{\text{hundred entries}}{\text{hour}}}$

(b)  $\frac{1}{8} \int_0^8 E(t) dt \approx \frac{1}{8} \left[ \frac{1}{2}(2)(0 + 4) + \frac{1}{2}(3)(4 + 13) + \frac{1}{2}(2)(13 + 21) + \frac{1}{2}(1)(21 + 23) \right] =$

$\boxed{10.6875}$  or 10.688

This approximates the average number of entries (in hundreds) in the box from noon until 8pm.

(c) Entries processed:  $P(t) = t^3 - 30t^2 + 298t - 976 \frac{\text{hundreds of entries}}{\text{hour}}$  for  $8 \leq t \leq 12$

Number of entries not yet processed at midnight is the number of entries at 8pm minus the number processed from 8pm until midnight:

$$E(8) - \int_8^{12} P(t) dt = 23 - 16 = \boxed{7 \text{ hundred entries}}$$

(d) The entries were processed most quickly at the absolute maximum of  $P(t)$  on  $8 \leq t \leq 12$  which will occur at  $t = 8$ , or  $t = 12$ , or at some critical point of  $P$  on the interval.

$$P'(t) = 3t^2 - 60t + 298 = 0 \Rightarrow t = 9.184 \text{ and } 10.817$$

$$P(8) = 0, \quad P(9.184) = 5.089, \quad P(10.817) = 2.911, \quad P(12) = 8$$

Hence, the entries were processed most quickly at  $\boxed{t = 12 \text{ or midnight}}$