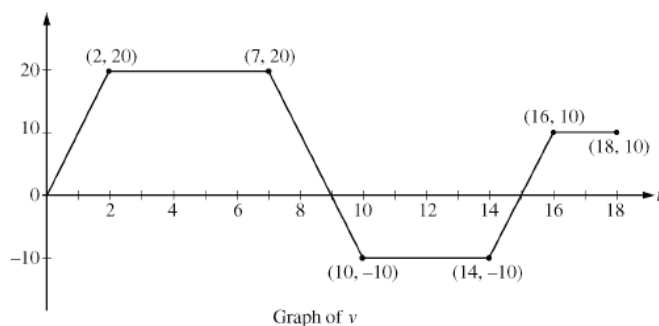


“MR. CALCULUS” ANSWERS TO THE 2010 FORM B FREE RESPONSE QUESTIONS

AB/BC 4



- (a) The squirrel changes direction at $t = 9$ and at $t = 15$, because its velocity changes sign at each of these points in time.
- (b) The squirrel is farthest from building A at the absolute maximum of $x(t)$ on $0 \leq t \leq 18$. This will occur at $t = 0$, $t = 18$, or at a critical point of $x(t)$ or where $x'(t)$ or $v(t) = 0$.
From (a), $v(9) = v(15) = 0$. $x(0) = 0$

$x(9) = x(0) + \int_0^9 v dt = 0 +$ the area of the trapezoid extending from $t = 0$ to $t = 9$. The area is $\frac{1}{2}(20)(9+5) = 140$, so the squirrel moves 140 units towards B during that time.

$$\text{So, } x(9) = x(0) + \int_0^9 v dt = 0 + 140 = 140$$

The area of the trapezoid from $t = 9$ to $t = 15$ is $\frac{1}{2}(10)(6+4) = 50$, so the squirrel moves back 50 units towards A during that time. The area of the trapezoid from $t = 15$ to $t = 18$ is $\frac{1}{2}(10)(3+2) = 25$, so the squirrel moves 25 units towards B during that time.

$$\text{So, } x(15) = x(9) + \int_9^{15} v dt = 140 - 50 = 90 \quad \text{And } x(18) = x(15) + \int_{15}^{18} v dt = 90 + 25 = 115$$

Hence, the squirrel is farthest from building A at $t = 9$ when it is 140 units away.

- (c) Based on the calculations in Part (b), the total distance traveled by the squirrel is $140 + 50 + 25 = 215$.

- (d)

$$a(t) = v'(t) = \frac{v(10) - v(7)}{10 - 7} = \frac{-10 - 20}{10 - 7} = -10 \Rightarrow a(t) = -10$$

$$v(t) - 0 = v'(9)(t - 9) \Rightarrow v(t) = -10(t - 9) = -10t + 90$$

$$x(t) = \int v(t) dt = -5t^2 + 90t + C \quad \text{From Part (b), } x(9) = 140$$

$$\Rightarrow -5 \cdot 9^2 + 90 \cdot 9 + C = 140 \Rightarrow C = -265 \Rightarrow x(t) = -5t^2 + 90t - 265$$