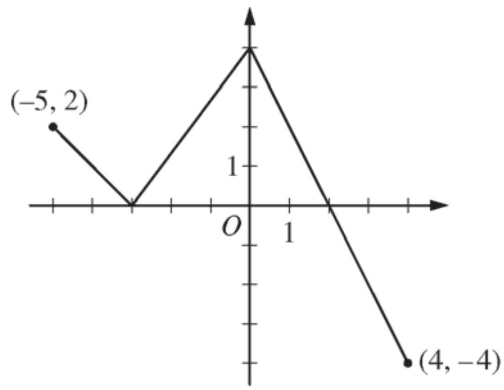
2014 AB/BC #3
(no calculator)



Graph of f

$g(x)=\int_{-3}^{x} f(t)\,dt \;\Rightarrow\; g'(x)=f(x)$ So this is also the graph of $g'(x)$.

(a) $g(3)=\int_{-3}^{3} f(t)\,dt=\int_{-3}^{2} f(t)\,dt+\int_{2}^{3} f(t)\,dt=\boxed{\frac{1}{2}(5)(4)-\frac{1}{2}(1)(2)}=9$

(The area of the $2nd$ triangle is subtracted since it is below the x-axis.)

(b)

g is increasing when g' or $f>0 \;\Rightarrow\; -5<x<2$

g is concave down when g'' or $f'<0 \;\Rightarrow\; g'$ or f is decreasing $\Rightarrow\; -5<x<-3$ and $0<x<4$

Both occur when $\boxed{-5<x<-3 \text{ and } 0<x<2}$

(c)

$h(x)=\frac{g(x)}{5x} \;\Rightarrow\; h'(x)=\frac{5xg'(x)-5g(x)}{25x^2}$ using the quotient rule

$h'(3)=\frac{5(3)g'(3)-5g(3)}{25(9)}=\frac{15f(3)-5(9)}{25(9)}=\boxed{\frac{15(-2)-5(9)}{25(9)}}=-\frac{1}{3}$

(d)

$p(x)=f\left(x^2-x\right) \;\Rightarrow\; p'(x)=(2x-1)f'\left(x^2-x\right)$

The slope of the tangent line to p at $x=-1$ is

$p'(-1)=\left(2(-1)-1\right)f'\left((-1)^2-(-1)\right)=(-3)f'(2)=\boxed{(-3)\left(\frac{-4-4}{4-0}\right)}=6$