2014 AB #5
(no calculator)

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

f and g are twice differentiable so both of them and their derivatives are continuous.

(a) The relative minimum of f on $[-2,3]$ occurs at $\boxed{x=1}$ because f' changes from negative to positive there.

(b) Because f' is continuous and differentiable on the interval, the Mean Value Theorem guarantees that there must be a value c on $(-1,1)$ such that

$$f''(c) = \frac{f'(1) - f'(-1)}{1-(-1)} = \frac{0-0}{2} = 0$$

(c) $h(x) = \ln(f(x)) \Rightarrow h'(x) = \dfrac{f'(x)}{f(x)} \Rightarrow h'(3) = \dfrac{f'(3)}{f(3)} = \boxed{\dfrac{1/2}{7}} = \dfrac{1}{14}$

(d) $\int_{-2}^{3} f'(g(x))g'(x)dx = f(g(x))\Big]_{-2}^{3} = f(g(3)) - f(g(-2)) = f(1) - f(-1) = \boxed{2-8} = -6$

The antiderivative above was evaluated by observation, but you could have used a substitution: $u = g(x)$ so $du = g'(x)dx$

So, the integral becomes $\int_{g(-2)}^{g(3)} f'(u)du = f(g(3)) - f(g(-2)) = -6$ as above