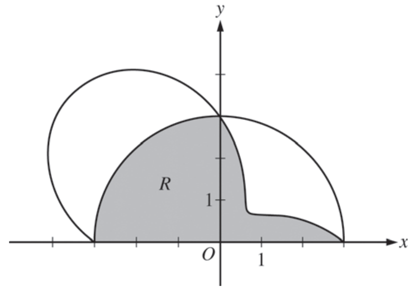
2014 BC #2
(calculator active)



The polar curves are circle $r_1 = 3$ and $r_2 = 3 - 2\sin(2\theta)$ for $0 \le \theta \le \pi$.

Note: $r_1 = r_2$ when $\theta = 0, \dfrac{\pi}{2}$, and π

(a) The area, R, inside of r_1 and inside of r_2 on this interval is

$$\frac{1}{2}\int_0^{\pi/2} r_2^{\,2}\, d\theta + \frac{1}{2}\int_{\pi/2}^{\pi} r_1^{\,2}\, d\theta \approx \boxed{9.707963268} = 9.708$$

OR

$$\frac{1}{2}\int_0^{\pi/2} r_2^{\,2}\, d\theta + \frac{1}{4}\pi(3)^2 = \boxed{2.639379797 + \frac{9\pi}{4}} = 9.708$$

(b)

$x = r_2 \cos\theta = (3 - 2\sin 2\theta)\cos\theta$

Using your calculator: $\left.\dfrac{dx}{d\theta}\right|_{\theta=\pi/6} \approx \boxed{-2.366025009} = -2.366$

(c)

The distance between the two curves on $\left(0, \dfrac{\pi}{2}\right)$ is $d = r_1 - r_2$.

We want $\left.\dfrac{d}{d\theta}(d)\right|_{\theta=\pi/3} = \boxed{-1.999998667} = -2$

(d)

$r = 3 - 2\sin 2\theta$ and $\dfrac{d\theta}{dt} = 3$

$$\frac{dr}{dt} = -2\cos 2\theta \bullet 2\frac{d\theta}{dt} \Rightarrow \left.\frac{dr}{dt}\right|_{\theta=\pi/6} = -2\cos\left(2\left(\frac{\pi}{6}\right)\right)\bullet 2(3) = -12\cos\left(\frac{\pi}{3}\right) = \boxed{-12\left(\frac{1}{2}\right)} = -6$$