2014 AB/BC #4
(no calculator)

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	−120	−150

(a) Average acceleration on [2,8] =

$$\frac{1}{8-2}\int_2^8 v'(t)\,dt = \frac{1}{6}\left(v(8)-v(2)\right) = \boxed{\frac{1}{6}\left(-120-100\right)} = -\frac{110}{3}\frac{\text{m}}{\text{min}^2}$$

(b) $\boxed{\text{Yes}}$ $v_A(t) = -100$ for some $t \in (5,8)$.
Since $v_A(t)$ is differentiable, then it is continuous on the interval.
Therefore the Intermediate Value Theorem guarantees that because – 100 is between 40 and – 120, there exists a t between 5 and 8 where $v_A(t) = -100$.

(c) $\int_2^{12} v(t)\,dt = x(12) - x(2) \quad \Rightarrow \quad x(12) = x(2) + \int_2^{12} v(t)\,dt = \boxed{300 + \int_2^{12} v(t)\,dt}$

Using a trapezoidal approximation for the definite integral:

$$x(12) \approx \boxed{300 + \frac{1}{2}(5-2)(100+40) + \frac{1}{2}(8-5)(40-120) + \frac{1}{2}(12-8)(-150-120)}$$

$= -150$ meters west of the Origin station

(d) Train A is traveling $x_A(t)$ meters east and train B is traveling $y_B(t)$ meters north, so the distance between them is the hypotenuse, $z(t)$, of a right triangle and $v_B(t) = y'_B(t) = -5t^2 + 60t + 25$. We are to find the rate that $z(t)$ is changing at time $t = 2$, or $z'(2)$.

$z^2(t) = x_A^2(t) + y_B^2(t) \Rightarrow$ so $z(2) = 500$ and, taking the derivative $\Rightarrow$

$2z(t)z'(t) = 2x_A(t)x'_A(t) + 2y_B(t)y'_B(t) = 2x_A(t)v_A(t) + 2y_B(t)v_B(t)$

$z(2)z'(2) = x_A(2)v_A(2) + y_B(2)v_B(2) \Rightarrow$

$$z'(2) = \boxed{\frac{300(100) + 400\left[-5(2)^2 + 60(2) + 25\right]}{500}} = 160\frac{\text{m}}{\text{min}}$$