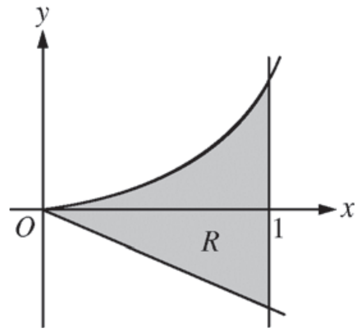
2014 BC #5
(no calculator)



Let $y_1 = xe^{x^2}$ and $y_2 = -2x$ in the graph above.

(a)

Area $= \int_0^1 (y_1 - y_2)\,dx = \frac{1}{2}e^{x^2} + x^2 \Big]_0^1 = \boxed{\left(\frac{1}{2}e + 1\right) - \left(\frac{1}{2} + 0\right)} = \frac{1}{2}e + \frac{1}{2}$

(b)

When rotated about $y = -2$ the volume obtained is

$$\boxed{\pi\int_0^1 \left[(y_1 - (-2))^2 - (y_2 - (-2))^2\right] dx}$$

(c)

The perimeter of R is the sum of the length of curves y_1 and y_2 and the vertical line from $y_1(1) = e$ to $y_2(1) = -2$:

$$\int_0^1 \sqrt{1 + \left(\frac{dy_1}{dx}\right)^2}\,dx + \int_0^1 \sqrt{1 + \left(\frac{dy_2}{dx}\right)^2}\,dx + e - (-2)$$

$\frac{dy_1}{dx} = 2x^2e^{x^2} + e^{x^2}$ and $\frac{dy_2}{dx} = -2$

$$\boxed{\int_0^1 \sqrt{1 + \left(2x^2e^{x^2} + e^{x^2}\right)^2}\,dx + \int_0^1 \sqrt{1 + (-2)^2}\,dx + e - (-2)}$$